Minimizing Timing Luck with Portfolio Tranching

*The Difference Between Hired and Fired*

February 2015
Abstract

While the choice of rebalance frequency is often well thought out, the choice of when to rebalance a portfolio is often an after-thought. Research papers, and even live strategies, typically use convenient calendar dates like the first or last trading days of the month.

In this paper, we introduce the concept of offset portfolios, the collection of portfolios running identical strategies with an identical rebalance frequency, but rebalancing on unique days. The variance in total return profile between these offset portfolios highlights the impact of timing luck: the deviation from the long-term expected strategy return due entirely to when a portfolio is rebalanced.

This variance can have a massive impact on both hypothetical back-tested research as well as live track records. We demonstrate that from the period of 1950-2014, the tactical trading methodology proposed by Faber (2013) may overstate its total return profile from the true expected strategy return by 1800 percentage points, simply due to its choice of end-of-month rebalancing.

Timing luck affects tactical, smart beta, and strategic portfolios alike. In this paper we provide examples of portfolios of each type, generate the offset portfolios, and build a model for the spread between the best and worst performing offset portfolios over time.

We then introduce the concept of portfolio tranching, proving that an equal-weight portfolio-of-offset-portfolios minimizes the impact of timing luck. We then demonstrate the impact tranching would have on each of the examples provided earlier in the paper.
**Introduction: What is *Timing Luck?***

Asset management due diligence places heavy emphasis on “Process and People”: the methodology behind how a portfolio is constructed and the qualifications of the people designing and executing that process. Process due diligence usually starts with high-level investment philosophy and works down to portfolio execution decisions.

“How often do you rebalance?” is a commonly asked question. The *frequency* with which a portfolio is rebalanced, $f$, is often well thought out, creating a balance between the quickness that the portfolio can adapt to new market dynamics with the desired turnover and tax efficiency of the portfolio.\(^2\) The date upon which most portfolios rebalance often lines up with a standard calendar time window: weekly, monthly, quarterly, et cetera. Strategies then usually rebalance on the first or last days of these periods. But robustness testing around this selection is infrequent at best. The choice of rebalancing date, however, can have a massive impact – potentially hundreds of basis points a year – on the total return profile of any strategy.

\(^2\) A portfolio which rebalances annually has $f = 1$; a portfolio which rebalances quarterly has $f = \frac{1}{4}$, as it rebalances four times a year.
The Impact of Timing Luck: A Simple Example

To demonstrate the impact that the rebalance date can have, we will construct a simple tactical trading strategy. The strategy will compare the price of the S&P 500 Index to its trailing 200-day moving average. The portfolio will be fully invested in the index when its price is above its 200-day moving average\(^3\), otherwise the portfolio will hold cash. The portfolio will reevaluate this rule every 21 trading days\(^4\).

With a 21-day rebalance frequency, 21 separate strategy equity curves can be calculated (portfolio 1 rebalances on the 1\(^{st}\) trading day of each month, portfolio 2 rebalances on 2\(^{nd}\) trading day, et cetera). We will call this set of possible portfolios the *offset portfolios*. The variance in total return results among the offset portfolios is demonstrated in the equity curves below:

\(^3\) A total return index is utilized in these calculations
\(^4\) For the convenience of calculations, instead of adhering to a strict calendar cycle, each portfolio is rebalanced on a 21 trading-day cycle.
The variance in total returns above is dependent entirely on *when* certain market events occurred within a given 21-day period. For example, consider the scenario where the market sharply sells off at the end of the month, causing the S&P 500 Index to plunge below its 200-day moving average, only to recover shortly thereafter at the beginning of the next month. In this scenario, only those strategies rebalancing near the end of the month would be affected by the whipsaw – the remainder would passively ride out the turbulence. How particular market events fall around a portfolio's rebalance frequency can have a significant impact on the total return profile: an effect we call *timing luck*.

To highlight the tremendous impact of timing luck, over the 65-year back-test of our example strategy, the spread between the total
return of the best performing equity curve and the worst is a 5800 percentage points. Yet the strategies are identical: this difference is accounted for only by when each portfolio was selected to rebalance.

<table>
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<tr>
<th>Rebalance Day</th>
<th>Growth of $10,000</th>
<th>Annualized Return</th>
<th>Annualized Volatility</th>
<th>Annualized Return to Volatility</th>
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In his massively popular paper, Faber (2013) proposes a similar strategy, using a 10-month moving average against the S&P 500 Index, rebalancing at month end. Recreating this strategy from 1950 to 2015, the portfolio returned approximately 8200% on a cumulative basis. However, if we estimate a month to be 21-days and compute all the
potential offset portfolios, we find that the average offset portfolio returned only approximately 6400% over this period. In other words, the timing luck of having selected the month-end date may have accounted for a total return excess of 1800 percentage points over the back-test period. That is not to say the methodology proposed by Faber (2013) is without merit, but rather that in the context of this test, the particular results reported were potentially 1.25 standard deviations above the true expected return of the strategy simply due to the rebalancing date.

Modeling Performance Variance Due to Timing Luck

Let us assume a simple model for a strategy’s return profile by decomposing returns into benchmark returns and active returns:

\[ r_s = \beta r_b + r_a \]

Where \( r_b \) is the benchmark return, \( \beta \) is the strategy’s sensitivity to the benchmark, and \( r_a \) is the active return profile.

Let us assume that the underlying returns are distributed normally and independently from one another:

\[ r_b \sim N(\mu_b, \sigma_b^2) \]

\[ r_a \sim N(\mu_a, \sigma_a^2) \]
It follows that:

\[ r_s \sim N(\beta \mu_b, \beta^2 \sigma_b^2) + N(\mu_a, \sigma_a^2) \]

Given two strategies that follow the same investment process, but rebalance with a strict offset to one another, we would calculate the difference between the strategies as:

\[ r_{s1} - r_{s2} = (\beta r_b + r_{a1}) - (\beta r_b + r_{a2}) = r_{a1} - r_{a2} \]

It follows that:

\[ r_{s1} - r_{s2} \sim N(\mu_{a1} - \mu_{a2}, \sigma_{a1}^2 + \sigma_{a2}^2 - 2\rho_{a1,a2} \sigma_{a1} \sigma_{a2}) \]

Since the strategies follow an identical investment process, we assume they have equal expected active returns and variances, leaving us with:

\[ r_{s1} - r_{s2} \sim N(0, 2\sigma_a^2 - 2\rho_{a1,a2} \sigma_{a}^2) \]

In other words, while the expected difference between these two strategies is zero, the variance is going to be driven by the correlation of their active returns.

We define the variance of timing luck between any two offset portfolios to be:

\[ 2\sigma_a^2 - 2\rho_{a1,a2} \sigma_{a}^2 \]
Estimating The Correlation of Active Returns

If we can estimate the correlation between the active returns of our offset portfolios, we can gain an understanding of the impact that timing luck will have without necessarily calculating every possible offset portfolio.

Given only the returns and allocations of a single offset portfolio, we propose a simple model we call correlation decay to gain a good estimate of correlation. Correlation decay is defined as:

$$\rho_{\text{decay}} = \Delta (1 - \rho)$$

Where $\Delta$ is annualized portfolio turnover and $\rho$ is the average correlation of the portfolio with itself before and after a rebalance.\(^5\)

Consider the simple tactical model previously calculated. Using one of the offset portfolios as our sample data, we estimate an annualized turnover rate of 146%. We also know that each time the portfolio turns over, it moves from fully invested to cash or vice versa. We expect that the correlation between these two portfolios to be zero. Therefore, our annualized $\rho_{\text{decay}}$ is equal to 146% – or a daily correlation decay of 0.57%.

\(^5\) $\rho$ can be calculated simply as the covariance between two portfolios – given portfolio weights $w_1$ and $w_2$ and full covariance matrix $\Sigma$ – is $w_1^T \Sigma w_2$. 
How is this metric useful? Using this decay, we can expect the portfolio that rebalances on the 1st trading day of the month versus the portfolio that rebalances on the 2nd will only be 99.43% correlated over the long run. The portfolio that rebalances on the 1st day versus one that rebalances on the 10th will be 94.30% correlated. We have to take particular note, however, of the circular nature of the rebalances: the portfolio that rebalances on the 1st versus the one that rebalances on the 21st are actually only a single day away and therefore will have a correlation of 99.43%. The overall correlation matrix would take the form:

\[
\begin{bmatrix}
100.0\% & 99.43\% & \cdots & 99.86\% & 99.43\% \\
99.43\% & 100\% & \cdots & 99.29\% & 99.86\% \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
99.86\% & 99.29\% & \cdots & 100\% & 99.43\% \\
99.43\% & 99.86\% & \cdots & 99.43\% & 100.0\%
\end{bmatrix}
\]

If we construct a matrix that contains how much we expect each offset portfolio to be correlated relative to each other, we can estimate a single correlation number as the mean of this matrix. In our above example, we calculate an expected correlation to be 97.01%.

It is important to note here that the entries in the table are constrained by the appropriate limits for correlation. We know that \( \Delta \) will be implicitly limited by the frequency with which a portfolio
rebalances, \( f \). Assuming no leverage, any individual portfolio turnover must be limited between \([0, 1]\). Therefore, with rebalance frequency \( f \), \( \Delta \) will be limited between \([0, \frac{1}{f}]\). Since \( \rho \) is limited between \([-1, 1]\), \((1 - \rho)\) will be limited between \([0, 2]\). With these limits, we can compute that the limits on annual \( \rho_{\text{decay}} \) will be between \([0, \frac{2}{f}]\). Since daily \( \rho_{\text{decay}} \), limited between \([0, \frac{2}{252f}]\), is applied only to the first \( \left\lfloor \frac{252}{2} f \right\rfloor \) offset portfolios, the entries in our correlation matrix are limited between \([-1, 1]\).

**Estimating the Impact of Timing Luck**

Using a single offset portfolio as our sample, we can calculate an estimate of active return variance against our benchmark and then calculate the annualized return variance between offset portfolios due to timing luck using the following formula:

\[
2\sigma_a^2 - 2\rho_{a1,a2} \sigma_a^2
\]

We calculate our sample’s annualized active return variance against the S&P 500 to be 1.175%. Using our prior computed estimate for correlation, we calculate an annualized volatility of timing luck of 2.65%.
The 265 basis points of volatility due to timing luck is \textit{not} additive to a traditional portfolio volatility measure. Rather, it captures the range around the \textit{true} expected annualized return that a sample annualized return of an offset portfolio may fall within. If our expected return for the strategy is 6.35\%, our three-year 95\% confidence range for sample annualized returns is 13.75\% to 24.35\%. That range may represent the difference between \textit{hired} and \textit{fired} – all due entirely to when we chose to rebalance.

We can extrapolate this volatility to analyze the sort of total return range we can expect between the best performing offset portfolio and the worst. In our previous example, over the 65-year back-test period ending December 2014, there was a realized spread within offset portfolio performance of over 4500 percentage points.
Timing Luck in Smart Beta

Tactical strategies are not the only portfolios that are susceptible to the incredible power of timing luck; any strategy that has a fixed rebalance frequency may be subject to large variations in total return due simply to when it rebalances.

“Smart Beta” portfolios are no different. A common formula for these portfolios is to use a fixed rebalance frequency and a fixed look-back period to construct a portfolio of securities based upon a factor tilt or other non-market capitalization weighted methodology. Here we address only a single, but popular, example: value. Due to the availability of data, portfolio construction will focus specifically on the U.S. Consumer Discretionary Sector between 3/1999 and 6/2013. A point-in-time database was utilized to reduce the potential impact of survivorship bias in the dataset.

For the value factor, securities within the universe are ranked based on their most current market-to-book value, with the lowest 25% being selected subject to a minimum holding of 10 securities. In this portfolio, securities are inversely weighted relative to their market-to-
book value and the entire portfolio is reconstituted annually. For this sample test, 12 offset portfolios are created, each rebalance offset from the prior by 21 days. Against, using the same process we plot a spread model and compare it against the realized spread.

It should come as no surprise that similar results are obtained for other factor portfolios, such as momentum and low volatility.

**Timing Luck in Strategy Portfolios**

Even many strategic portfolios prove to be susceptible to timing luck; in particular those portfolios that use historic market dynamics as an input to their construction methodology. Traditionally these portfolios break asset-class returns into monthly samples and generate relevant statistics off of these samples. However, as we will show, *when*
those samples are taken can have a significant impact on the resulting portfolio construction.

One method of generating a strategic portfolio is to use trailing realized asset class returns to estimate a multivariate distribution, used as input to an optimization process. In this example we use the trailing 756 trading days of data broken into equal 21-day chunks – a choice that is akin to using the trailing 3 years broken into monthly periods. From these samples we generate sample expected returns for each asset and a sample-variance covariance matrix. Using these statistics, we can generate an efficient frontier.

Unfortunately, efficient frontier construction via naïve mean-variance optimization is incredibly sensitive to estimates of the expected return. In effort to account for this, we can use an alternate construction methodology that assumes all asset classes have an equivalent Sharpe ratio, which we assume to be 0.5. This allows us to utilize volatilities, which tend to be more stable, to back out expected return estimates that will, in turn, make efficient frontier construction more stable.

Given the choice to use 21-day chunks, there are 21 possible days to begin the process, creating 21 efficient frontiers. In our example, we
utilize 31 ETFs representing global assets from style-based equities to different fixed-income sectors.

While these efficient frontiers appear tightly bundled, their variance can result in significantly different recommended portfolios. Consider, for example, the allocations recommended by each frontier for a portfolio targeting the maximum expected return for a 6% volatility level:
While there is moderate continuity from one offset sample portfolio to the next, as the offset distance expands (the length between rebalance days), the portfolio allocations begin to more significantly diverge. This result is likely indicative that results are clustered around a few outlier market events having significant impact on volatility and correlation estimates.

The above portfolios represent just a single snapshot in time. Using a walk-forward process, we can construct the 21 offset portfolios. Each portfolio is constructed utilizing an identical methodology but is rebalanced on a different date. The equity curves below highlight the impact of volatility from timing luck on performance results:
Over the backtest period, the best performing portfolio has a total return of 31.31% while the worst has a total return of 21.86%. This near-1000 basis point total return difference can be attributed entirely to when the portfolios rebalanced.

**Proving Portfolio Tranching Minimizes the Volatility of Timing Luck**

Now that we have demonstrated the significant impact that the date of rebalancing can have upon both a portfolio’s construction and its total return profile, the question is, “what can we do about it?” Mathematically, we want to find a method through which we can construct a portfolio that minimizes the volatility due to timing luck. We
propose that one such solution is portfolio tranching, whereby a strategy invests equally across all of its offset portfolios.

As before, we assume that each offset portfolio has the same expected return and volatility profile. For convenience – but without loss of generality – we also assume that these expected returns and volatilities are both equal to one. This assumption implies that our previously constructed correlation matrix – which we used to estimate our average correlation between offset portfolios – is the variance-covariance matrix, $\Sigma$, for our offset portfolios.

Since the offset portfolios are assumed to have equal volatility, any excess volatility in a portfolio of offset portfolios is due entirely to timing luck. Therefore, solving for a minimum volatility portfolio will minimize its impact. The solution to such a portfolio is:

$$\bar{w} = \frac{\Sigma^{-1} \bar{1}}{\bar{1}^T \Sigma^{-1} \bar{1}}$$

Where $w$ is the vector of solution weights and $\bar{1}$ is an Nx1 vector of 1s and N is the number of offset portfolios. It should be noted that although we assumed that expected return for each offset portfolio was equal to 1, the expected return is not actually necessary to solve for this portfolio.
The solution to this equation is trivial due to the unique nature of $\Sigma$. Specifically, $\Sigma$ is *symmetric circulant*: a square matrix where each row vector is rotated one element to the right of the preceding row vector. A special property of such a matrix is that its inverse – in this case $\Sigma^{-1}$ – is also symmetric circulant.

This property guarantees that the result of the operation $\Sigma^{-1}\vec{1}$ will be equivalent to $k\vec{1}$ for some constant $k$. The simple intuition behind this result is that since the rows all contain the same values, multiplying by a vector of ones (the operation for row summation), results in a vector of equivalent constants. Therefore, we can re-write our solution as:

$$w = \frac{k\vec{1}}{\vec{1}^T\vec{1}}$$

Since the $k$’s cancel out, we are left with a vector of ones divided by $\vec{1}^T\vec{1}$, which is equal to the number of offset portfolios. *In other words, our minimum variance solution is an equal-weight portfolio.*

**Portfolio Tranching versus a Tactical Model**

Using our prior 200 day moving average tactical timing model example, we can test the impact that tranching has on total return. In a purely
theoretical model, we would use all 21 offset portfolios and rebalance back to equal-weight daily. Unfortunately, in the real world, this is not a pragmatic solution. Therefore, we will use four equally spaced offset portfolios (e.g. offset portfolios #1, #6, #11 and #16) and rebalance back to equal weight on a weekly basis.

Below we plot the individual offset portfolios (light gray), the average offset portfolio (black), and the portfolio constructed using the tranching methodology (orange) described above. We can see that even using just four of the offset portfolios by and large eliminates any significant deviation from the set average.

![Equity Curves of Tactical Offset Portfolios, Their Average, and a Tranched Portfolio](image)

Most convenient about this methodology is that operationally we need not actually treat construction like a portfolio-of-portfolios. Rather, we can estimate an equal-weight exposure to each offset portfolio by
constructing a single portfolio on a weekly basis that is merely the
average of the underlying weights of portfolios generated over the
current and prior three weeks. *In practice, this is how Newfound
Research implements tranching within its own portfolios.*

**Portfolio Tranching for a Factor Model**

The identical methodology can be applied to reduce the impact of timing
luck in the factor portfolios we developed above. As an example, we will
revisit the Value Factor Consumer Discretionary portfolio, which
exhibited the largest volatility of timing luck of all the factor portfolios
at a whopping annualized 4.30%. For this factor, we constructed 12
individual offset portfolios, each of which was rebalanced on an annual
basis and offset from the prior by 21 days. We construct the tranched
portfolio by rebalancing to an equal weight position among the 12 offset
portfolios on a monthly basis. Once again, as seen in the plotted equity
curves below, the tranched portfolio (orange) closely tracks the average
offset portfolio (black), almost entirely muting the impact of timing luck
(variance) seen in the offset portfolios (gray).
Portfolio Tranching for Strategic Asset Allocation

It should come as little surprise that tranching works equally well within a strategic portfolio context. A tranching methodology may actually provide potential benefits beyond limiting timing luck. Since the expected return and variance-covariance samples utilized to construct each offset portfolio will be slightly different from one another, averaging them together is akin to resampling. Below, we plot the example allocations over time for one of the offset portfolios we built in the strategic asset allocation example discussed before:
In comparison to the single offset portfolio, we can see in the allocations of the tranched portfolio – using *all* of the offset portfolios we created before – that the averaging process helps reduce short-term portfolio churn and turnover, increase diversification across assets, and limits extreme exposures due to outlier events that affect sampled data.
Conclusion

While the frequency of rebalancing is a common topic for due diligence of an investment strategy, the date of rebalancing often goes ignored.

In this paper, we introduce the concept of offset portfolios: portfolios generated by an identical strategy with identical rebalancing frequency, but whose date of rebalance is offset by a consistent period of time. For example, a set of portfolios rebalanced monthly, but with one rebalanced on the 1\textsuperscript{st} of the month, another on the 2\textsuperscript{nd}, another on the 3\textsuperscript{rd}, et cetera. In the context of tactical, factor, and strategic portfolios, we utilize these offset portfolios to demonstrate the massive impact that rebalance timing luck can have on total return performance.

We propose a framework based on turnover and correlation changes to estimate the potential impact of timing luck when only a single sample portfolio is available. This framework allows for the construction of a 3-standard deviation model spread, allowing us to estimate the maximum performance to expect between identical strategies that are rebalanced on different days.

We then introduce the concept of portfolio tranching: a simple methodology based on constructing a portfolio-of-portfolios of the
individual offset portfolios. Utilizing the concept of minimal-variance portfolios, we prove that an equal-weight methodology maximally reduces the impact of timing luck. Finally, we demonstrate the impact that tranching has on reducing timing luck in the construction of tactical, factor, and strategic portfolios.

Timing luck has massive implications for performance evaluation. Ultimately, unless accounted for, results demonstrated on both a back-tested as well as a live basis may be massively skewed by this factor. The effects are so powerful that two managers, following identical strategies, may have markedly different performance results over time. One may be bestowed with praise for their alpha-generating abilities and the other may be fired for perceived underperformance, based solely on a rebalance date. For many asset managers, timing luck may be the hidden factor that leads to the difference between hired and fired.